Thermal transport of the solar captured dark matter and its impact on the indirect dark matter search

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We study the thermal transport occurring in the system of solar captured dark matter (DM) and explore its impact on the DM indirect search signal. We particularly focus on the scenario of self-interacting DM (SIDM). The flows of energies in and out of the system are caused by solar captures via DM-nucleon and DM-DM scatterings, the energy dissipation via DM annihilation, and the heat exchange between DM and solar nuclei. We examine the DM temperature evolution and demonstrate that the DM temperature can be higher than the core temperature of the Sun if the DM-nucleon cross section is sufficiently small such that the energy flow due to DM self-interaction becomes relatively important. We argue that the correct DM temperature should be used for accurately predicting the DM annihilation rate, which is relevant to the DM indirect detection.

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Dark matter (DM) composes about 25% of the energy density of in the universe and plays an important role in the structure formation. It was shown that if the galactic halo is constituted by weakly interacting massive particles (WIMPs), there is a high possibility that these WIMPs are captured by the Sun [1–8, 10]. In general there are thermal energy flows between the captured DMs and the nuclei in the Sun. Microscopically, such flows are caused by particle scatterings. For collisionless cold dark matter (CCDM), the scatterings are only between the DM and solar nuclei. Regarding the huge difference in abundance between the two, it is reasonable to take the DM temperature to be identical to the core temperature of the Sun. However, the scenario of SIDM can change the picture dramatically. In such a scenario, the energy transports via DM self-capture and DM-nucleus scattering compete with each other. In particular, if DM-nucleus interaction is much weaker than expected, the thermal exchange between DMs and nuclei would be much less efficient. As a result, the DM temperature can be distinct from the solar core temperature.

It is worth mentioning that the DM abundance is not much affected with a suppressed DM-nucleon cross section $\sigma_{\chi p}$ provided SIDM is considered [11]. The accumulated DM abundance is determined by the balance among the capture, the evaporation, and the annihilation rates. Earlier works on such processes only consider DM-nucleon interactions. The new effects from DM self-interaction are investigated recently [8–11]. The consideration of SIDM comes from the discrepancies between the numerical N-body simulations using the hypothesis of CCDM and the astrophysical observations on the

small structure of the universe [12]. The CCDM simulations [13] predict cuspy profiles in the center regions of galaxies, which conflict with flatten cores found in our Milky Way (MW) [14], other nearby dwarfs [15], and low luminous galaxies [16, 17]. There are additional puzzles concerning the sizes of subhalos. The observed MW satellites are hosted by much less massive subhalos compared to sizes of the most massive subhalos arising from simulations. The absence of such massive DM subhalos is referred to as the "too big to fail" problem in the galaxy formation. The existence of SIDM (in particular those involving the characteristic velocity-dependence cross sections [18–20]) is one of the solutions to alleviate these inconsistencies.

In this paper, we investigate the thermal energy transport between the trapped DM and the nuclei in the Sun, as well as the energy flows due to DM captures. With DM self-interactions taken into account, the DM number trapped in the Sun evolves according to

$$\frac{dN_\chi}{dt} = C_c + C_s N_\chi - C_a N_\chi^2 \ , \tag{1}$$

while the energy flows in and out of the system of trapped DM in the Sun are governed by

$$\frac{d\left(N_\chi E_\chi(t)\right)}{dt} = J_c + (J_\chi + J_s)N_\chi - J_a N_\chi^2 \ . \tag{2}$$

The coefficients $C_{c,s}$ are referred to as the DM capture rates due to DM-nucleon scattering cross section $\sigma_{\chi p}$ and DM-DM scattering cross section $\sigma_{\chi\chi}$, respectively. The coefficient C_a is related to DM annihilation rate in the Sun. In this work, we shall focus on the self-interaction dominant scenario with $C_s^2 \gg 4C_c C_a$ [8]. The left hand side of Eq. (2) is the total kinetic energy of the trapped DM with $E_\chi(t)$ the average kinetic energy of an individual DM. The factors $J_{c,s,a}$ are thermal transport coefficients corresponding to coefficients C_c, C_s, and C_a, respectively. The coefficient J_χ describes the heat exchange between trapped DM and the nuclei in the Sun.

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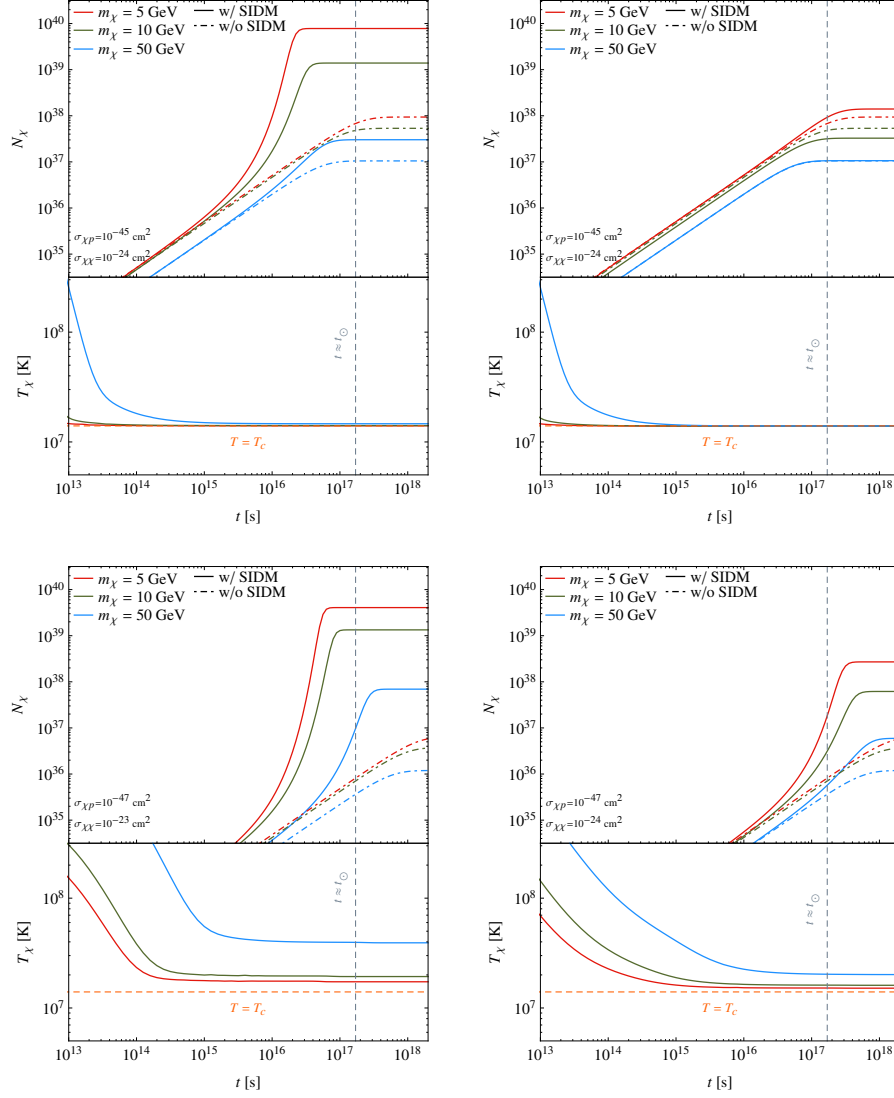†glin@cc.nctu.edu.tw
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FIG. 1: The time evolutions of N_χ and T_χ for $\sigma_{\chi p} = 10^{-45}$ cm^2 (upper pannel) and $\sigma_{\chi p} = 10^{-47}$ cm^2 (lower pannel) with $\sigma_{\chi\chi} = 10^{-23}$ cm^2 (left) and 10^{-24} cm^2 (right).

We do not concern the mass range where DM evaporation rate is significant. In this case, the DM abundance in the Sun is severely reduced so that the signal strength of DM annihilation is suppressed.

Since we aim at studying the temperature evolution of the trapped DM, it is important to compare mean collision time between a pair of trapped DMs and that between a trapped DM and nucleus in the Sun. We make the comparison for the cases of spin-independent (SI) and spin-dependent (SD) DM-nuclei scattering cross sections, respectively. We first note that the mean collision time between two DMs in the Sun is

$$\tau_{\chi\chi}(t) \simeq \frac{V_\odot}{N_\chi(t)\sigma_{\chi\chi}\bar{v}}, \tag{3}$$

while the mean collision time between DM and nucleus in the Sun is

$$\tau_{\chi\odot} \simeq \frac{V_\odot}{\sum_i N_i \sigma_{\chi A_i}^{\mathrm{SI}} \bar{v}} \tag{4}$$

for the case of spin-independent cross section with $V_\odot$ the solar volume, $\bar{v}$ the average velocity of trapped DMs, N_i the number of nucleus i in the Sun, and $\sigma_{\chi A_i}^{\mathrm{SI}}$ the spin-independent DM-nucleus scattering cross section. We neglect possible numerical factors on the right hand side of Eqs. (3) and (4) for this order of magnitude estimation. The time scale τ_χ^{eq} for DMs in the Sun to reach thermal equilibrium can be estimated by the condition $\tau_\chi^{\mathrm{eq}} \simeq \tau_{\chi\chi}(\tau_\chi^{\mathrm{eq}})$. To solve this approximated equation, we first assume that $N_\chi(\tau_\chi^{\mathrm{eq}})$ is still far from the maximal value of N_χ and verify this assumption later. In this case, one can show that $N_\chi(\tau_\chi^{\mathrm{eq}}) = C_c\tau_\chi^{\mathrm{eq}}$ for $C_s^2 \gg 4C_cC_a$.

With this input, we obtain $\tau_\chi^{\rm eq} = \sqrt{V_\odot / C_c \sigma_{\chi\chi} \bar{v}}$.

For further discussions, we shall take $m_\chi = 10$ GeV as a benchmark. For such a DM mass, we have $\sum_i N_i \sigma_{\chi A_i}^{\rm SI} \simeq 40 N_H \sigma_{\chi p}^{\rm SI}$ by assuming isospin invariant DM-nucleon couplings. Hence we may write $r \equiv \tau_\chi^{\rm eq} / \tau_{\chi\odot} = 40 N_H \sigma_{\chi p}^{\rm SI} / \sigma_{\chi\chi} N_\chi(\tau_\chi^{\rm eq})$. To simplify this relation further, we note that the average mass density of hydrogen in the Sun is roughly 1 g/cm^3. In other words, $N_H \simeq 6 \times 10^{53}$ given the volume of the Sun approximately at 10^{33} cm^3. Using this value of N_H and the relation $N_\chi(\tau_\chi^{\rm eq}) = \sqrt{V_\odot C_c / \sigma_{\chi\chi} \bar{v}}$ with $C_c \simeq 5.4 \times 10^{65} (\sigma_{\chi p}^{\rm SI}/{\rm cm}^2){\rm s}^{-1}$ and $\bar{v} \simeq 900$ km/s, we obtain $r \simeq 10^9 \sqrt{\sigma_{\chi p}^{\rm SI} / \sigma_{\chi\chi}}$. We are interested in the parameter range that gives $r < 1$, i.e., $\sigma_{\chi p}^{\rm SI} / \sigma_{\chi\chi} < 10^{-18}$. In this case, the trapped DMs reach to the thermal equilibrium among themselves before the energy exchange between the trapped DMs and the surrounding nuclei becoming efficient. Besides the condition $r < 1$, we need to ensure the ratio $C_s^2 / 4 C_c C_a \equiv 1.9 \times 10^3 (\sigma_{\chi\chi}/\sigma_{\chi p}^{\rm SI})(\sigma_{\chi\chi}/{\rm cm}^2)$ is much greater than unity for the consistency of our argument. The cross section combination $(\sigma_{\chi p}^{\rm SI}, \sigma_{\chi\chi}) = (10^{-45}{\rm cm}^2, 10^{-23}{\rm cm}^2)$ is an example of satisfying both $r < 1$ and $C_s^2 / 4 C_c C_a \gg 1$. With these parameters, we have $\tau_\chi^{\rm eq} = 4.5 \times 10^{13}$ s, which is much shorter than the age of the Sun, $\tau_\odot \approx 10^{17}$ s. For spin-dependent cross section, we found $r \simeq 1.8 \times 10^8 \sqrt{\sigma_{\chi p}^{\rm SD} / \sigma_{\chi\chi}}$ and $C_s^2 / 4 C_c C_a = 10^5 \times (\sigma_{\chi\chi}/\sigma_{\chi p}^{\rm SD})(\sigma_{\chi\chi}/{\rm cm}^2)$. Since $C_c \simeq 10^{64} \times (\sigma_{\chi p}^{\rm SD}/{\rm cm}^2){\rm s}^{-1}$, the slightly different cross section combination $(\sigma_{\chi p}^{\rm SD}, \sigma_{\chi\chi}) = (5 \times 10^{-44}{\rm cm}^2, 10^{-23}{\rm cm}^2)$ gives the same $\tau_\chi^{\rm eq}$ as the previous spin-independent case.

The above discussions justify the thermal equilibrium state of DM in the early stage of capture for certain combinations of $\sigma_{\chi p}$[1] and $\sigma_{\chi\chi}$. We can write $E_\chi(t) = s k_B T_\chi(t)/2$ in Eq. (2) where s is the degree of freedom of each DM. We recapitulate the meaning of various coefficients in Eq. (2). The coefficient J_c describes the energy flow due to DM capture caused by DM-nucleus scattering, J_χ describes the energy exchange between DMs and nuclei, J_s is related to the energy flow due to DM capture caused by DM self-interaction, and J_a is related to the energy dissipation due to DM annihilation. All the quantities are positive except J_χ which depends on the difference between the solar core temperature T_c and the DM temperature T_χ. In particular, $J_\chi < 0$ when $T_\chi > T_c$.

When the Sun sweeps across the MW halo, there are collisions between DM and the solar nuclei. If the DM velocity is smaller than the solar escape velocity after the collision, the DM will be gravitationally bounded within the Sun. The velocity of an infalling DM at the shell

with radius r inside the Sun is $w = \sqrt{u^2 + v_{\rm esc}^2(r)}$. Here u is the DM velocity in the halo and $v_{\rm esc}^2(r)$ is the solar escape velocity at such shell. The kinetic energy of the trapped DM with an average over the energy loss in the capture process is given by

$$\bar{E} = \frac{m_\chi}{4} \left(\frac{m_\chi - m_{\rm A}}{m_\chi + m_{\rm A}} \right)^2 u^2 + \frac{m_\chi}{2} \frac{(m_\chi^2 + m_{\rm A}^2)}{(m_\chi + m_{\rm A})^2} v_{\rm esc}^2(r), \tag{5}$$

where m_χ is the DM mass and $m_{\rm A}$ is the mass of nuclei A. We have $\bar{E} = m_\chi v_{\rm esc}^2(r)/4$ if it is due to DM-DM scattering, i.e., $m_{\rm A} = m_\chi$. We note that the collisions among DMs redistribute the DM kinetic energies such that the average DM kinetic energy becomes $E_\chi(t)$ in Eq. (2) when the thermal equilibrium is reached. The relation between $\bar{E}$ and $E_\chi(t)$ will be discussed later. The energy flow per shell volume is

$$\frac{dJ_c}{dV} = \int n_{\rm A} \sigma_{\chi\rm A} v_{\rm esc}^2(r) \frac{f(u)}{u}$$
$$\times \left[1 - \frac{(m_\chi - m_{\rm A})^2}{4 m_\chi m_{\rm A}} \frac{u^2}{v_{\rm esc}^2(r)} \right] \bar{E} du , \tag{6}$$

where $n_{\rm A}$ and $\sigma_{\chi\rm A}$ are the nuclei number density and the DM-nuclei cross section, respectively. The DM velocity in the halo is assumed to follow Maxwell-Boltzmann distribution, i.e.,

$$f(x) = \sqrt{\frac{6}{\pi}} \frac{\rho_o}{m_\chi \bar{v}} x^2 e^{-x^2} e^{-\eta^2} \frac{\sinh(2x\eta)}{x\eta} , \tag{7}$$

where $x^2 = 3(u/\bar{v})^2/2$ and $\eta^2 = 3(v_\odot/\bar{v})^2/2$ with $\bar{v} \approx 270$ kms^{-1} the DM dispersion velocity in the halo and $v_\odot = 220$ kms^{-1} the relative velocity between the Sun and the MW. The density ρ_0 is the DM local density[2] taken to be 0.3 GeV/cm^3. The DM capture by Sun's nuclei gives

$$J_c = \xi \sum_{\rm A} b_{\rm A} \frac{(m_\chi^2 + m_{\rm A}^2)}{(m_\chi + m_{\rm A})^2} \left(\frac{\sigma_{\chi\rm A}}{\rm pb} \right) \langle \phi_{\rm A}^2 \rangle, \tag{8}$$

where $b_{\rm A}$ is the number fraction of nuclei A in the Sun and $\langle \phi_{\rm A}^2 \rangle$ is the average gravitational potential square contributed by nuclei A. The quantity ξ is given by

$$\xi \equiv \sqrt{\frac{3}{8}} N_\odot \rho_0 \frac{v_{\rm esc}(R_\odot)}{\bar{v}} v_{\rm esc}^3(R_\odot) \frac{{\rm erf}(\eta)}{\eta}$$
$$\approx 1.2 \times 10^{23}\,{\rm GeV\,s}^{-1} \left(\frac{\rho_0}{0.3\,{\rm GeV/cm}^3} \right) \left(\frac{270\,{\rm km/s}}{\bar{v}} \right). \tag{9}$$

Similarly, the energy flow due to self-capture J_s can be derived by making the replacements $m_{\rm A} \to m_\chi$ and

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[1] In the subsequent discussions we shall only focus on spin-independent cross section since it is better constrained by direct detection experiments. We drop the superscript SI henceforth for simplicity.

[2] A slightly larger local DM density was derived in Ref. [21].

$n_A \rightarrow n_\chi$ (n_χ is the DM number density in the Sun). We obtain

$$J_s \approx \sqrt{\frac{3}{32}} \rho_0 \sigma_{\chi\chi} \frac{\text{erf}(\eta)}{\eta} \frac{v_{\text{esc}}(R_\odot)}{\bar{v}} v_{\text{esc}}^3(R_\odot) \langle \phi_\chi \rangle^2 . \quad (10)$$

Since DMs are populated within a spherical region with a radius $R \simeq 0.1 R_\odot$, we can take $\langle \phi_\chi^2 \rangle \approx \langle \phi_\chi \rangle^2$ where $\langle \phi_\chi \rangle = 5.1$ is the average gravitational potential of the DM.

The energy of captured DM could be dissipated due to annihilation. The energy flow due to this process is

$$J_a = \frac{\int 4\pi r^2 n_\chi^2(r) E_\chi(t) dr}{(\int 4\pi r^2 n_\chi(r) dr)^2} \langle \sigma v \rangle \quad (11)$$

with $\langle \sigma v \rangle$ the thermally-averaged DM annihilation cross section. Numerically, we have

$$J_a(t) \approx 7.5 \times 10^{-65} \, \text{GeV s}^{-1} \left(\frac{s m_\chi}{10 \, \text{GeV}} \right)^{3/2} \left(\frac{E_\chi(t)}{\text{GeV}} \right)^{-1/2},$$
$$(12)$$

where s is the degree of freedom of each DM. We shall take $s = 3$ for the numerical analysis later.

Finally, the captured DMs continuously exchange energies with the solar nuclei. The energy transport due to collisions between single DM and the surrounding nuclei in the Sun is given by [3]

$$\begin{aligned} J_\chi &= 8\sqrt{\frac{2}{\pi}} \rho_c m_\chi \frac{k_B(T_\odot - T_\chi)}{(m_\chi + m_A)^2} \\ &\times \sum_A f_A \sigma_{\chi A} \left(\frac{m_A k_B T_\chi + m_\chi k_B T_\odot}{m_\chi m_A} \right)^{1/2}, \end{aligned}$$
$$(13)$$

where $\rho_c \approx 110$ g/cm^3 is the core density of the Sun, f_A is the mass fraction of nuclei A. We note that the above relation is derived by taking $s = 3$. We have numerically solved Eqs. (1) and (2). Both equations run from $t_0 = 10^{13}$ s, which is roughly the time scale for DMs in the Sun to thermalize according to our earlier arguments. As a first approximation, the initial condition for $E_\chi(t)$ at $t_0 = 10^{13}$ s is taken to be $\bar{E}$ in Eq. (5) with u^2 and r averaged. We point out that $\bar{E}$ is the DM average kinetic energy before the thermalization. When DMs reach to the thermal equilibrium, they are populated more closely to the solar core. Hence one expects $E_\chi(t_0) > \bar{E}$ by the conservation of mechanical energy, since the gravitational potential energy of the thermalized DM is in general smaller than that of DM before thermalization. Nevertheless $E_\chi(t_0)$ and $\bar{E}$ differ within a factor of two, and in fact the DM temperature in the current epoch is independent of $E_\chi(t_0)$ as we shall argue later.

The evolutions of DM number and DM temperature are presented in Fig. 1. We take $\sigma_{\chi p} = 10^{-45}$, 10^{-47} cm^2 and $\sigma_{\chi\chi} = 10^{-23}$, 10^{-24} cm^2 for $m_\chi = 5$, 10, 50 GeV, respectively as the benchmark points. It is clearly seen

that N_χ at $t \simeq 10^{13}$ s is indeed far below the maximal N_χ. This justifies our assumption earlier for deriving τ_χ^{eq}. We also see that the DM accumulation is enhanced by DM self interactions. In particular, the N_χ enhancement is more significant for smaller m_χ and it is very sensitive to the ratio $\sigma_{\chi\chi}/\sigma_{\chi p}$.

Fig. 1 shows that DM temperature is higher than the temperature of solar core in the early stage for different DM masses. The DM temperature T_χ drops with time due to energy flow from DMs to solar nuclei. However, T_χ is not always approaching to T_c. Those cases with $\sigma_{\chi p} = 10^{-47}$ cm^2 show such a behavior. Particularly, $T_\chi/T_c \sim 3.5$ for $\sigma_{\chi\chi} = 10^{-23}$ cm^2 and $m_\chi = 50$ GeV in the current epoch. We note that $J_a N_\chi^2$ caused by the DM annihilation does not affect the DM temperature. Hence DM temperature evolution is controlled by parameters J_c, C_c, J_s, C_s and J_χ. However, as N_χ accumulates, the effect by J_c and C_c become negligible. We thus have

$$\frac{dE_\chi(t)}{dt} \approx J_\chi + J_s - C_s E_\chi(t). \quad (14)$$

We observe that $E_\chi(t)$ or $T_\chi(t)$ approaches to a constant value when the right hand side of Eq. (14) vanishes. This implies $J_\chi \rightarrow 0$, i.e., $T_\chi \rightarrow T_c$, if the self-interaction induced coefficients J_s and C_s are negligible. On the other hand, T_χ does not approach to T_c if DM self-interaction cannot be neglected. We note that the gap between T_χ and T_c increases for heavier DM. This is because J_χ scales as $1/m_\chi$ for heavy DM. Thus, to balance $J_s - C_s E_\chi(t)$, $T_\chi - T_c$ must be enhanced to compensate the $1/m_\chi$ suppression in J_χ. We argue that the DM temperature T_χ in the current epoch is fixed once the values for $\sigma_{\chi\chi}$, $\sigma_{\chi p}$ and m_χ are given. Essentially, $T_\chi(t_\odot)$ is the solution to $J_\chi + J_s - C_s E_\chi(t_\odot) = 0$ with J_χ the function of $\sigma_{\chi p}$, m_χ and $E_\chi(t)$, and both J_s and C_s the functions of $\sigma_{\chi\chi}$. Clearly the initial condition $E_\chi(t_0)$ does not affect $T_\chi(t_\odot)$. Indeed we have varied $E_\chi(t_0)$ within a reasonable range in our numerical analysis and found that $E_\chi(t)$ ($T_\chi(t)$) approaches to the same fixed point.

We remark that the thermal equilibrium time scale τ_χ^{eq}, which we choose as t_0, is obtained by a rough estimation. To study the effect from such an uncertainty, we also solve Eqs. (1) and (2) with $t_0 = 10^{14}$ s. We find that $T_\chi(t_\odot)$ remains the same.

It is important to note that N_χ depends on the DM temperature T_χ. This is due to the dependence of N_χ on C_a. Hence the indirect detection signal could also be sensitive to T_χ. We compare the annihilate rate $\Gamma_A \equiv C_a N_\chi^2/2$ calculated with precise T_χ with that computed with the assumption $T_\chi = T_c$. The result is presented in Fig. 2. DM temperature affects both N_χ and C_a in Γ_A. With $T_\chi > T_c$, we have $C_a(T_\chi) < C_a(T_c)$ because $C_a \propto T_\chi^{-3/2}$ [5]. As a result, we can easily see that $N_\chi(T_\chi) > N_\chi(T_c)$ from Eq. (1). As long as the enhancement on N_χ^2 can overwhelm the suppression on C_a, the annihilation rate Γ_A would be enhanced by adopting the precise DM temperature T_χ. In the limit $C_s^2 \gg 4 C_c C_a$, $\Gamma_A \rightarrow C_s^2/2C_a$ when N_χ in the Sun reaches to the maxi-
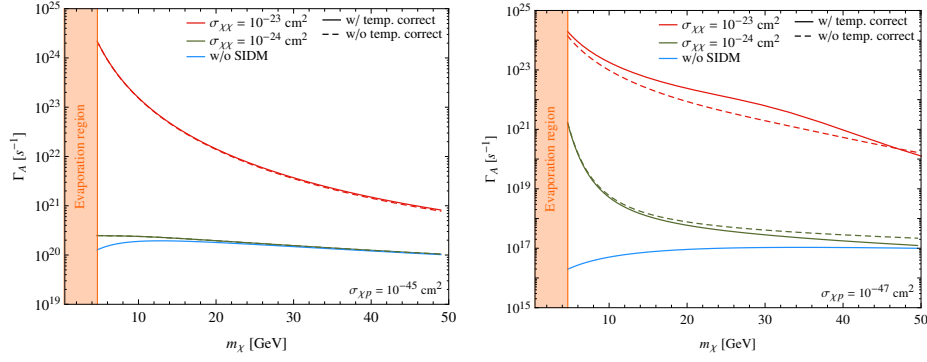
FIG. 2: Total annihilation rate Γ_A for $\sigma_{\chi p} = 10^{-45}$ cm^2 and 10^{-47} cm^2 with $\sigma_{\chi\chi} = 10^{-23}$ cm^2 and 10^{-24} cm^2. Dashed lines are those results calculated with $T_\chi = T_c$.

mum. One has $\Gamma_A(T_\chi) > \Gamma_A(T_c)$ as a result. This is seen from the right panel of Fig. 2 with $\sigma_{\chi p} = 10^{-47}$ cm^2 and $\sigma_{\chi\chi} = 10^{-23}$ cm^2. However, with $\sigma_{\chi\chi} = 10^{-24}$ cm^2 and $\sigma_{\chi p}$ kept the same, N_χ is still growing, i.e., $N_\chi \simeq C_c t_\odot$. Hence $\Gamma_A \simeq C_a C_c^2 t_\odot^2/2$, which implies $\Gamma_A(T_\chi) < \Gamma_A(T_c)$. We presented such a scenario in the right panel of Fig. 2 as well.

We have taken the Sun as an example to illustrate the possible temperature difference between DMs and their surrounding medium. We stress that the derivation in this paper is generally applicable to other massive celestial objects. The only required information is the DM dispersion velocity, the DM local density, and physical properties of the celestial object. In summary, we have derived and solved the thermal transport equation for DMs trapped in the Sun for the first time. In the

SIDM scenario, the DM temperature and the core temperature of the Sun could be different. We have shown that DM annihilation rate is sensitive to the DM temperature. Hence it is imperative to adopt the precise DM temperature to calculate the indirect DM event rates.

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